Dublin City Council
Samuel Beckett Bridge

The Samuel Beckett Bridge, which was designed by Santiago Calatrava, is a cabled-stayed cantilever bridge with a span of 120 metres crossing the river Liffey in Dublin city. Its single support is 28 m from the South quay and the bridge can be turned through 90° to facilitate marine traffic. The shape resembles a harp on its side.

Beams and Cantilevers

A beam bridge is essentially a rigid structure supported at both ends. Various techniques are used to maximise strength while minimising weight. A girder bridge can be thought of as a variation of the beam bridge. I-beam and box steel girders are commonly used in construction; these shapes are much less prone to bending. Girder bridges can span more than 200 m.

Suspension Bridges

When a large distance, such as a river or estuary, has to be bridged in a single span a suspension bridge is usually preferred. Typically two large supporting cables (called ‘suspenders’) are anchored at each side and fixed to the tops of the towers. The rest of the structure is suspended from these two cables.

Cable-Stayed Cantilever Bridges

These differ from suspension bridges in the following ways:
• there is no single suspender cable
• the suspension cables are attached to the tower(s) and to the deck and are not vertical
• the deck is attached to the tower(s)
• there may be only a single tower or pylon.

The Structure of the Samuel Beckett Bridge

The bridge deck (bridge beam) has a box-girder structure which provides rigidity. The load is transferred and dissipated (spread) throughout the structure of the bridge deck and into the cable-stays. The cable-stays are in tension and the load is transferred to the pylon which in turn transfers the load to the bridge’s foundation, and in turn to the ground. The pylon is in compression.

When a load is applied to a member of the bridge structure, that member will deform to some extent and when this deformation takes place, internal forces in the material of the structural member will resist it. These internal forces are called stresses. The force transmitted across a section of the structural member divided by the area of that section gives the intensity of the stress; it therefore has the same units as pressure – pascals (Pa) or N/m².

Balance

In the case of a structure that rotates around a single support it is desirable to eliminate unbalanced forces. This is usually done by adding a suitable counterweight. (Cranes used on building sites usually have concrete counterweights attached to the short end of the rotating beam.)
The shorter end of the Samuel Beckett Bridge has almost 2500 tonnes of steel, lead and heavy concrete added to it so that the centre of gravity of the whole structure is directly above the pivot. The bridge is then in equilibrium.

**Rotating a Bridge**

A swing bridge rotates horizontally about a vertical axis. The minimum power needed to rotate the bridge depends on the mass distribution (the moment of inertia) and the maximum time of rotation. A braking system is required to slow the rate of rotation and bring the bridge to rest at specified points.

**Power Requirements**

The diagram (Fig. 7) represents a bridge of uniform mass that can rotate about one end.

For simplicity let us assume the following:

Length ($l$) 100 metre
Mass ($m$) 1000 tonne
Turning time 60 seconds
(for simplicity we assume
30 s acceleration and
30 s deceleration).

At the half way point (45° or 0.785 radian) the structure has its maximum angular velocity and maximum energy. The average angular velocity is 0.785 rad in 30 s and the maximum is twice this value (0.052 rad/s).

The rotational energy ($E$) is related to the angular velocity ($\omega$) and the moment of inertia ($I$) as follows: $E = \frac{1}{2} I \omega^2$. (The moment of inertia of a uniform beam that is rotating about one end is given by $I = \frac{1}{3} m l^2$, where $m$ is its mass and $l$ its length.). So the maximum energy is $\frac{1}{2} m l^2 \omega^2$, or about 4.5 MJ. Since this energy is accumulated over 30 s the average power is 4.5/30 MJ/s, i.e. 150 kJ/s or 150 kW. If the torque is uniform then the acceleration is uniform and more power is required when the bridge is approaching its maximum velocity; it would in fact be twice the average power, i.e. 300 kW.

If the bridge were twice as long or if it were turned twice as quickly then the average power required would be four times as great; the energy is proportional to $l^2$ and $\omega^2$. All these estimates neglect frictional forces.

**Rotating Balanced Structures**

The structure above is unbalanced.

A balanced structure could be made

- by moving the axis of rotation to the centre or
- by adding a counterweight while maintaining the same overall length as shown in the diagram.

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**Dublin City Council**

**Dublin City Council** is the democratic body governing Dublin City. It is the largest Local Authority in the country with a staff of approximately 6,200 and it provides a wide range of services for its citizens and businesses.

The key executive in the council is the City Manager who has overall responsibility for all functions of the council. The Council also has an elected assembly of 52 members, elected every five years. The Lord Mayor is elected by the assembly to act as its chairman and as the symbolic head of the city.

The Council has a number of committees focusing on the development of policy in six major areas of Council activity. One third of the members are from outside the Council and represent, for example, social partner, business or community interests. There are also five committees focusing on the needs of specific areas.

The principal Council services (by expenditure) are environmental protection, housing and building, water supply and sewerage, road transportation and safety.

Dublin City covers an area of 11,500 hectares and has a population of 500,000. The total annual Council expenditure is nearly €800m.

The **Samuel Beckett Bridge** was designed by the internationally acclaimed Engineer and Architect, Dr Santiago Calatrava Valls. The bridge will be a key link in the City’s street network and should facilitate the continued integration and development of the Dublin Dockland’s area.

*You can find out more on this and other Dublin City Council lessons on www.sta.ie*
Dublin City Council  
Samuel Beckett Bridge  
Teaching Notes  

Syllabus References  
The relevant syllabus references are:  

**Leaving Certificate Physics (Syllabus pp. 25-31)**  
Moments Simple experiments with a number of weights. Appropriate calculations. (Only problems involving co-planar, parallel forces need be considered.)  
Conditions for equilibrium Vector sum of the forces in any direction is zero. The sum of the moments about any point is zero.  

**Leaving Certificate Applied Mathematics**  
Circular Motion and SHM Circular motion and simple harmonic motion (SHM).  
Moments of Inertia Moment of inertia of rigid bodies about various axes  

**Leaving Certificate Technology (pp. 12 - 17)**  
The Design Process; Project and Quality Management; Materials and Production  

Learning Outcomes  
On completion of this lesson, students should be able to  
- Distinguish between beam, truss, cantilever and suspension bridges  
- Understand why I-beam and box girders are commonly used in construction  
- Understand the function of the cables in a cable-stayed cantilever bridge  
- Appreciate the importance of balancing rotating structures  
- Understand the calculation of the power required to rotate a uniform beam.  

General Learning Points  
The following can be used to revise the main learning points in the lesson.  
- A simple beam bridge is a rigid structure supported at both ends.  
- A cantilever is a beam that projects beyond its supporting structure. The Samuel Beckett Bridge is a cabled-stayed cantilever bridge.  
- I-beam and box girders are much more rigid than simple beams of the same weight and length.  
- In a cable-stayed cantilever bridge the cables are in tension and the pylon is in compression.  
- Weights are added to the shorter end of the bridge until the centre of gravity is directly over the centre of rotation.  
- The power required to turn a uniform beam is proportional to its mass, its length squared and the angular velocity squared.
Dublin City Council
Samuel Beckett Bridge
Exercises

Student Activity

1. Cut a sheet of A4 paper into four strips of 297 × 52 mm.
   Fold one of the strips to form a triangular beam and glue the overlapped pieces. Place it across a gap between two blocks or tables that are 275 mm apart. What is the maximum weight it can support without falling?
2. Repeat the procedure with the other three strips of paper trying different shapes – square, rectangular or circular. Record your findings.
3. Find the mass of a ruler. Find the mass of a euro. Place the euro on one end of the ruler. Calculate the position of the centre of gravity. Verify your answer.
4. Design and make a truss bridge from one sheet of A4 paper. It should span 450 mm and should be free standing. What is the maximum weight it can support without falling? Record your design and findings. (It is possible to make a bridge to these specifications that can support over 0.5 kg or a 500 ml bottle of water.)

True/False Questions

a) An I-beam girder has a square cross-section. T F
b) The Vikings constructed steel truss bridges. T F
c) A cantilever is a horizontal beam that protrudes beyond the support point. T F
d) The deck of a cable-stayed cantilever bridge hangs from suspension cables. T F
e) A truss structure is much stronger than a simple beam of the same mass and length and of the same material. T F
f) The pylon of a cable-stayed cantilever bridge is in compression and the cables are in tension. T F
g) When a beam is balanced then the weight on each side is always the same. T F
h) The counterweight used in the Samuel Beckett Bridge is 250 tonnes. T F
i) The intensity of a stress has the same units as pressure, N/m². T F
j) The moment of inertia of a simple beam is proportional to the square of the length. T F

Check your answers to these questions on www.sta.ie

Examination Questions

Leaving Cert Physics (HL) 2007, Q. 1
A student investigated the laws of equilibrium for a set of co-planar forces acting on a metre stick. The student found that the centre of gravity of the metre stick was at the 50.4 cm mark and its weight was 1.2 N.

How did the student find (i) the centre of gravity, (ii) the weight, of the metre stick? Why is the centre of gravity of the metre stick not at the 50.0 cm mark?

The student applied vertical forces to the metre stick and adjusted them until the metre stick was in equilibrium. How did the student know that the metre stick was in equilibrium?

The student recorded the following data.

<table>
<thead>
<tr>
<th>position on metre stick /cm</th>
<th>11.5</th>
<th>26.2</th>
<th>38.3</th>
<th>70.4</th>
<th>80.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>magnitude of force /N</td>
<td>2.0</td>
<td>4.5</td>
<td>3.0</td>
<td>5.7</td>
<td>4.0</td>
</tr>
<tr>
<td>direction of force</td>
<td>down</td>
<td>up</td>
<td>down</td>
<td>up</td>
<td>down</td>
</tr>
</tbody>
</table>

Calculate:

(i) the net force acting on the metre stick
(ii) the total clockwise moment about a vertical axis of the metre stick
(iii) the total anti-clockwise moment about a vertical axis of the metre stick.

Use these results to verify the laws of equilibrium.

Leaving Cert Applied Mathematics (HL) 2007, Q. 8

Prove that the moment of inertia of a uniform square lamina, of mass \(m\) and side \(2r\), about an axis through its centre parallel to one of the sides is \(\frac{1}{12}m r^2\).

Leaving Cert Applied Mathematics (HL) 2006, Q. 8

(a) Prove that the moment of inertia of a uniform rod of mass \(m\) and length \(2\ell\) about an axis through its centre perpendicular to the rod is \(\frac{1}{3}m \ell^2\).

(b) A uniform rod of mass \(3m\) and length 1.2 metres can turn freely in a vertical plane about a horizontal axis through one end.

The rod oscillates through an angle of 120°.

(i) Find the angular velocity of the rod when the rod is vertical.
(ii) Find, in terms of \(m\), the vertical thrust on the axis when the rod is vertical.
Did You Know?

Stone and concrete are relatively weak under bending forces but extremely strong under compression. Stone bridges are often constructed using arches; the individual stones throughout the arch are all under compression and the more weight is applied to the bridge the stronger it will be. Such bridges have stood for thousands of years.

The first arch bridge made of iron was built at Telford in England in 1779. It is made of cast iron (as distinct from wrought or forged iron). As the mass production of steel advanced during the nineteenth century truss bridges became more favoured designs. During the twentieth century the development of very strong steel cables made it possible to construct suspension and cable-stayed cantilever bridges.

Biographical Notes

Santiago Calatrava Valls

Santiago Calatrava was born in Benimámet, Valencia, Spain in 1951. Having completed architectural studies in 1975 he studied civil engineering in Zurich until 1981.

His early work on bridges and train stations demonstrated his exceptional skill as an architect and engineer – an unusual combination. His designs are often inspired by the natural world.

His Communications Tower in Barcelona brought him international acclaim. This spectacular tower was begun in 1989 and completed in 1992.

In 2003 the James Joyce Bridge was opened in Dublin; this 40 m bridge was also designed by Calatrava.

Revise The Terms

Can you recall the meaning of the following terms?

acceleration, angular velocity, axis, beam bridge, box steel girders, cantilever, cable-stayed cantilever, counterweight, deceleration, energy, equilibrium, girder, I-beam girder, kW, lead, mass, MJ, moment of inertia, N/m², pascal (Pa), power, pylon, radian, steel, stress, suspension bridge, tonne, triangulation

Check the Glossary of Terms for this lesson on [www.sta.ie](http://www.sta.ie)